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Estimation of the loss factor of viscoelastic laminated panels from finite element analysis

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ABSTRACT

A wave finite element (WFE) method is applied for predicting wave dispersion, wave attenuation and dissipation in viscoelastic laminated panels. The method involves postprocessing (using periodic structure theory) of element matrices of a small segment of the structure, which is modelled using a stack of three-dimensional finite elements meshed through the cross-section. Each layer can be discretised using either one solid element or more solid elements in order to more accurately represent interlaminar stress and strain. The finite element model of the segment of the structure is typically very small, resulting in very small computation cost. Formulations for the evaluation of the global loss factor using the WFE approach are given. In particular a formulation to calculate the average loss factor in the general case of an anisotropic component is proposed. Numerical examples are then shown. These concern the evaluation of the dispersion curves and of the global loss factor for damped laminated panels of different constructions.

1. Introduction

Passive damping treatments are widely used in engineering applications to reduce noise radiation, amplitude of vibrations and risk of fatigue failure. Passive damping of large metal or fibre composite panels is accomplished typically by applying one of the following techniques: (i) a damping polymeric layer is attached to the outer surface of the host structure – referred to as unconstrained layer damping (ULD) treatment; (ii) a damping polymeric layer is either sandwiched between two layers of the structure or attached to the host structure and constrained by an additional thin layer – referred to as constrained layer damping (CLD) treatment. Multiple constrained damping layers can also be used in order to improve damping properties over a wider frequency and temperature range.

CLD treatment is generally more effective in dissipating energy than the correspondent ULD configuration. A classical analysis of CLD was carried out more than 40 years ago by Kerwin [1]. After this work, Ungar and Kerwin [2] gave a formulation for the loss factor in terms of energy, which has become the basis for the evaluation of the loss factor and the parametric design of damped composite structures. Since then, a number of works have been published to predict damping and to improve the performance of passive constrained layer treatments applied to beams and plates. Most of these works rely on assumptions and approximations concerning the stress–strain distribution in the solid based on the classical theory of sandwich beams, e.g. [3,4]. Since the dominant mechanism of damping in laminates and CLD treatments is shearing, it is important that the model accurately represents through-thickness strains and stresses. Extended theories,

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which consider longitudinal inertia, rotatory inertia, shear deformation and thickness deformation in the viscoelastic layer were proposed, for example, in [5–7]. However, these assumptions break down as the thickness, frequency and the complexity of the cross-section increase. This is particularly true if composite laminated structures must be analysed. In the context of analytical approaches, this can be achieved by assuming higher-order shear deformation theories, or considering a full three-dimensional elasticity theory. However, this leads to very complicated equations which are nearly intractable.

The most commonly employed tool for vibration analysis is the finite element (FE) method. However, although computer capacity has improved enormously, many structures, in particular in automotive, space, aeronautical and naval applications, are large and there are limits to the number of degrees of freedom (DOFs) the FE model of the whole structure might have. Therefore, most of the FE models are realised using two-dimensional shell elements. This results in models whose ability to represent the damping mechanism is often poor and inaccurate. Energy dissipation in CLD treatments is in fact predominantly due to the interlaminar shear that the viscoelastic layer undergoes during the vibration of the whole structure. Higher order shell elements could be used to overcome the problem, but developing reliable higher order shell elements is difficult. The use of solid elements could be the best choice in many cases, and these have substantially more DOFs resulting in impractically large FE models, in particular at high frequencies. Consequently, methods which can capture the three-dimensional through-thickness displacements and stresses, and provide fast and accurate approaches to predict damping and vibroacoustic behaviour of laminate panels are highly desiderable.

Amongst methods proposed in literature, a spectral finite element method was used by Shorter in [8] to study wave propagation and damping in viscoelastic laminates. The global loss factor was calculated using the strain energy method [2] in the context of a wave approach. In [8], the damping loss factor for an aluminium beam with CLD treatment and a typical automotive glass laminate were predicted considering frequency dependent material properties for the viscoealstic layer. In [9], Mace et al. studied wave propagation in the same sandwich panel analysed by Shorter [8] using a wave finite element formulation for one-dimensional waveguides. Ghinet and Atalla [10] obtained the global loss factor associated with the bending wave type in thick composite laminate plates with linear viscoelastic damping. They used a discrete laminate approach assuming each layer is described by a Reissner–Mindlin displacement field. A strain energy method was again used to estimate the damping loss factor for each wave type. Cotoni et al. [11] evaluated the global loss factor of the first wave propagating in a honeycomb sandwich panel in the context of finite element and periodic structures using the same approach proposed in the present paper. The loss factor was calculated considering the formulation given in [8]. Recently, Plagianakos and Saravanos [12] proposed a high-order finite element methodology for predicting modal damping and dynamic behaviour of composite plates of arbitrary lamination. In [12], the through-thickness displacement field in each layer of the structure was described using quadratic and cubic polynomials.

The main aim of this paper is to present an application of a wave finite element (WFE) method for two-dimensional structures [13] for predicting damping in viscoelastic laminate panels of arbitrary complexity. In particular the paper provides a fast, efficient and automated technique for the identification of dispersion, attenuation and damping. Basically, the WFE method is a numerical approach to evaluate wave propagation in two-dimensional homogeneous structures, whose properties can vary in an arbitrary manner through the thickness. The method involves conventional finite element analysis of a small segment of the structure. Typically, this consists of a stack of solid elements meshed through the cross-section. Each layer can be discretised using either one solid element or more solid elements in order to accurately represent interlaminar stress. Hence an accurate three-dimensional stress–strain state of the structure (in the context of FEA) is modelled. This enables the shear distribution to be correctly represented and the energy dissipation to be accurately evaluated, in particular in soft layers and when constrained layer damping treatments are applied. The mass and stiffness matrices of this small segment are then post-processed according to the theory of wave propagation in periodic structures [14], resulting in an eigenproblem, whose solutions yield the dispersion curves and the wave modes, which are related to the cross-section motion. It is worth pointing out that the method can equally be applied to curved panels but utilising the WFE approach described in [15] instead of that for flat panels [13]. One of the main advantages of the method consists in exploiting the full power of commercial FE packages. Therefore complicated constructions can be easily analysed, and the extensive element libraries of commercial FE tools can be used.

In Section 2 a short overview of the WFE method is shown and formulations for estimating the global loss factor using the WFE approach are presented. In particular a formulation for estimating the average loss factor in the general case of an anisotropic component is proposed. Section 3 contains numerical examples. The first example concerns an isotropic plate with CLD treatment, for which results for the damping loss factor are compared with results obtained by other authors. Further cases of viscoelastic laminated composite plates are then considered. Dispersion curves, attenuation and global damping loss factor as a function of frequency and of the heading direction, and the average loss factor are provided.

2. Estimation of the loss factor using WFE

A WFE method for two-dimensional structures as outlined in [13] is applied for predicting damping and attenuation in composite structures with arbitrary lay-up. Consider a composite laminate with an arbitrary stacking sequence, whose layers are made of linear elastic or viscoelastic materials. A small rectangular segment of sides L_x and L_y is taken from the structure and meshed, through the thickness, using conventional FE methods as shown in Fig. 1. Typically the FE model is

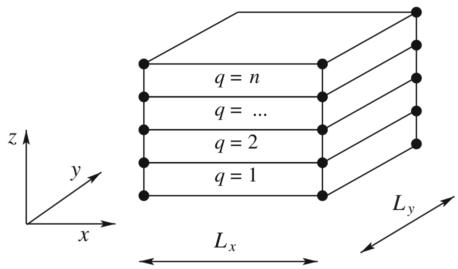

Fig. 1. FE model of a small rectangular segment.

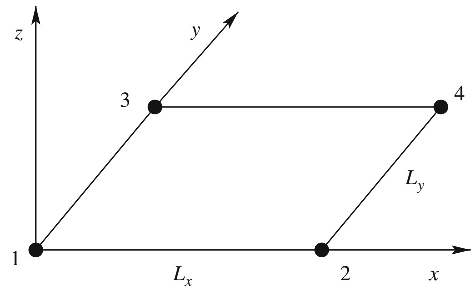

Fig. 2. Rectangular 2-D superelement.

realised using three-dimensional finite elements, defined by eight nodes having three degrees of freedom at each node, respectively, translation in the x, y and z directions. Each layer can be modelled using one or more three-dimensional solid elements. At the corners of the segment, the DOFs and nodal forces through the thickness of the segment are concatenated into vectors $\mathbf{q}_i$ and $\mathbf{f}_i$, $i=1,\ldots,4$ in order to define a 4-noded superelement as shown in Fig. 2. Interior and edge nodes can also be included as described in [13].

The DOFs and nodal forces of the segment are then ordered in the following way:

$$\mathbf{q}=[\mathbf{q}_1^{\mathrm{T}}\ \mathbf{q}_2^{\mathrm{T}}\ \mathbf{q}_3^{\mathrm{T}}\ \mathbf{q}_4^{\mathrm{T}}]^{\mathrm{T}},$$

$$\mathbf{f}=[\mathbf{f}_1^{\mathrm{T}}\ \mathbf{f}_2^{\mathrm{T}}\ \mathbf{f}_3^{\mathrm{T}}\ \mathbf{f}_4^{\mathrm{T}}]^{\mathrm{T}}, \tag{1}$$

where the superscript T denotes the transpose.

Structural damping is assumed, and for time harmonic motion the viscoelastic properties of each layer are characterised by complex components in the stiffness matrix. The properties of viscoelastic materials are influenced by many parameters, viz. frequency, temperature, static pre-load, aging and so on. Among these, frequency and temperature are the most important. In the following analysis it is assumed that the material properties are given and known. For example, viscoelastic properties of layers of polymeric material may be obtained using temperature-frequency charts supplied by manufacturers.

The mass $\mathbf{M}$ and the stiffness $\mathbf{K}$ matrices of the FE model are found typically using commercial FE packages. Considering frequency and temperature dependent material properties, the global stiffness matrix $\mathbf{K}$ of the FE model is given by

$$\mathbf{K}(\omega,T)=\mathbf{K}'+\mathrm{i}\mathbf{K}''=\sum_{q=1}^{n}\mathbf{K}_q'(\omega,T)+\mathrm{i}\mathbf{K}_q''(\omega,T), \tag{2}$$

where n is the total number of solid elements used in the FE discretisation while $\mathbf{K}_q'$ and $\mathbf{K}_q''$ are the real and imaginary stiffness matrix contributions of the qth finite element to the global matrices $\mathbf{K}'$ and $\mathbf{K}''$. The fact that the material properties are not constant is a difficulty. In principle, the stiffness matrix should be re-evaluated at each frequency and temperature (or reduced frequency) of interest. In order to avoid repeated evaluation of the stiffness matrix, assuming the Poisson ratio is independent of frequency, the stiffness matrix can be evaluated according to

$$\mathbf{K}_q(\omega,T)=\frac{E(\omega,T)}{E(\omega_0,T_0)}\mathbf{K}_q'(\omega_0,T_0)[1+\mathrm{i}\eta_q(\omega,T)], \tag{3}$$

where $\eta_q(\omega,T)$ is the material loss factor of element q, and $\mathbf{K}_q'(\omega_0,T_0)$ is the stiffness matrix of element q evaluated at a reference frequency and temperature ω_0,T_0. In this case the stiffness matrix $\mathbf{K}_q(\omega_0,T_0)$ needs only be computed once, and then scaled for other frequencies and temperatures.

The WFE method for two-dimensional structures as outlined in [13] is then applied. Details are not given here. Consider a plane wave propagating in a two-dimensional structure according to

$$w(x,y,z,t) = W(z)\mathrm{e}^{\mathrm{i}(-k_x x - k_y y + \omega t)}, \tag{4}$$

where $W(z)$ is the wave mode, ω is the angular frequency and $k_x = k\cos\theta$ and $k_y = k\sin\theta$ are the components of the wave vector $\mathbf{k}$ in the x and y directions. Under the passage of the wave, the nodal DOFs are related by periodicity conditions [14]

$$\mathbf{q}_2 = \lambda_x \mathbf{q}_1,\ \ \mathbf{q}_3 = \lambda_y \mathbf{q}_1,\ \ \mathbf{q}_4 = \lambda_x\lambda_y \mathbf{q}_1, \tag{5}$$

where

$$\lambda_x = \mathrm{e}^{-\mathrm{i}k_x L_x}, \quad \lambda_y = \mathrm{e}^{-\mathrm{i}k_y L_y}, \tag{6}$$

while equilibrium at corner 1 of the segment implies

$$[\mathbf{I}\ \ \lambda_x^{-1}\mathbf{I}\ \ \lambda_y^{-1}\mathbf{I}\ \ (\lambda_x\lambda_y)^{-1}\mathbf{I}]\mathbf{f} = \mathbf{0}. \tag{7}$$

The equation of motion of the segment

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{q} = \mathbf{f} \tag{8}$$

is projected onto the DOFs $\mathbf{q}_1$ using Eqs. (5) and (7), providing a WFE reduced eigenvalue problem relating $\mu_x = k_x L_x$, $\mu_y = k_y L_y$ and ω

$$[\overline{\mathbf{K}}(\mu_x,\mu_y) - \omega^2 \overline{\mathbf{M}}(\mu_x,\mu_y)]\mathbf{q}_1 = \mathbf{0}. \tag{9}$$

Eq. (9) relates the propagation constants μ_x and μ_y and the frequency ω. The solutions of Eq. (9) yield the dispersion equations, and the eigenvectors define the wave mode shapes.

Waves propagating in viscoelastic media are characterised by complex frequency or complex wavenumbers according to whether time or spatial attenuation is involved. In particular, harmonically forced wave propagation in a damped material leads to real frequencies and complex wavenumbers, which correspond to damped propagating, nearfield and oscillating but highly decaying waves. For the remainder of this paper, attention is focused on propagating waves, which are defined to be those waves which would be propagating (i.e. real wavenumber) in the undamped structure. These propagate energy over relatively large distances.

2.1. Frequency prescribed: time-harmonically excited structure

If the structure is excited harmonically, and the direction of propagation θ is specified, the WFE prediction of the wavenumber k and the wave mode $\mathbf{q}_1$ is obtained for each frequency from a polynomial or a transcendental eigenvalue problem, Eqs. (24)–(26) and (27)–(29) in [13].

In FEA, an often used approach to calculate the global loss factor for laminated viscoelastic structures is the modal strain energy method, first suggested by Ungar and Kerwin in [2]. An approximate formulation for estimating the modal loss factor using the modal strain energy in FEA, can be obtained as shown in [16]. The loss factor can also be calculated using the wave mode shapes. In the context of wave propagation, the global loss factor, associated with the jth propagating wave at frequency ω and heading θ, is expressed in terms of the FE stiffness matrices and the wave modes as

$$\eta_j(\omega,T,\theta) = \frac{\mathbf{V}_j^* \mathbf{K}''(\omega,T)\mathbf{V}_j}{\mathbf{V}_j^* \mathbf{K}'(\omega,T)\mathbf{V}_j}, \tag{10}$$

where $*$ denotes the conjugate transpose. $\mathbf{V}_j$ is obtained from the wave mode $\mathbf{q}_{1j}$ and the components of the wavenumber k_{xj} and k_{yj} using periodicity conditions (5)

$$\mathbf{V}_j = [\mathbf{q}_{1j}^{\mathrm{T}}\ \ \mathrm{e}^{-\mathrm{i}k_{xj}L_x}\mathbf{q}_{1j}^{\mathrm{T}}\ \ \mathrm{e}^{-\mathrm{i}k_{yj}L_y}\mathbf{q}_{1j}^{\mathrm{T}}\ \ \mathrm{e}^{-\mathrm{i}k_{xj}L_x}\mathrm{e}^{-\mathrm{i}k_{yj}L_y}\mathbf{q}_{1j}^{\mathrm{T}}]^{\mathrm{T}}. \tag{11}$$

Note that the wave modes, defined by the eigenvectors $\mathbf{q}_{1j}$, and $\mathbf{V}_j$ are complex, arising from the solution to the complex eigenvalue problem of Eq. (9).

2.2. Wavenumber prescribed: external space-harmonic initial conditions

If external space-harmonic initial conditions are applied, spatial decay is prevented, and transient oscillating but decaying vibrations are generated. The wavenumber vector is thus real and specified, and Eq. (9) is then solved for complex $\omega^2 = (\omega_r + \mathrm{i}\omega_i)^2$. The loss factor is then

$$\eta_j(\omega,T,\theta) = 2\frac{\omega_r\omega_i}{(\omega_r^2 - \omega_i^2)}. \tag{12}$$

Formulations (10) and (12) allow the evaluation of the loss factor for different propagation directions and for any frequency. Comparison between results obtained from (10) and (12) has shown that both formulations give very nearly the same prediction of the loss factor, e.g. Fig. 6.

2.3. Average loss factor

If the material of the laminate plate is isotropic, Eqs. (10) and (12) are independent of θ, and the global loss factor is spatially uniform. However, in fibre composite laminates, the wavenumber and damping depend on the heading direction. Hence, the global loss factor is a function of the propagation direction, and the dependence on θ in Eqs. (10) and (12) should be taken into account. Damping also depends upon the stacking sequences and modes of vibration. In these cases, a prediction of an average distribution of the damping loss factor is useful to evaluate damping characteristics for various external and environmental conditions. Such an average can be regarded as being taken over all propagation directions θ, or over a group of modes of vibration. In the following, a formulation to evaluate the average loss factor in terms of heading direction θ is proposed.

For the case of a diffuse field, when all waves are carrying the same energy, Cotoni et al. [11] showed that the average loss factor can be calculated by taking an average over the dispersion contour curves within a frequency band $\Delta\omega$ of interest. These dispersion contour curves are the dispersion relation of the form $\omega = f(k_x,k_y)$, that is the constant frequency contour in the wavenumber plane, i.e. the (k_x,k_y) plane. Fig. 8 shows an example of the dispersion contour curves for laminated plates. The average

$$\overline{\eta}(\omega,T) = \frac{\int_0^{2\pi} \eta(\omega,T)\,\mathrm{d}\theta}{\int_0^{2\pi} \mathrm{d}\theta} \tag{13}$$

could be taken. However, this average is generally not appropriate because waves have different velocities in different directions. Thus the loss factor varies from mode to mode. For this purpose, it can be useful to consider the number of resonant modes below a certain frequency, that is the mode count $N(\omega)$. The mode count of several structural elements have been investigated by many authors, and expressions for the mode count can be obtained theoretically, e.g. [17–20].

Consider the number of modes $\mathrm{d}N(\omega,\theta)$ associated with a small area $\mathrm{d}A$ spanned between θ and $\theta+\mathrm{d}\theta$, and enclosed by the contour curves at ω and $\omega+\mathrm{d}\omega$ in the wavenumber plane as shown in Fig. 3. We can assume that the loss factor associated with these modes is $\eta(\omega,T,\theta)$, which can be evaluated using Eq. (10) or (12). Therefore, an expression for the average loss factor is given by

$$\overline{\eta}(\omega,T) = \frac{\int \eta(\omega,T,\theta)\,\mathrm{d}N(\omega,\theta)}{\int \mathrm{d}N(\omega,\theta)}. \tag{14}$$

At sufficiently high frequency, the number of modes in a certain frequency band $\mathrm{d}\omega$ can be considered independent of the boundary conditions, and an asymptotic expression can be obtained. For example, for a two–dimensional component, the area of the wavenumber plane associated with each mode can be approximated by π^2/S, where S is the area of the structure [17]. Therefore, the asymptotic number of resonant modes $\mathrm{d}N(\omega,\theta)$ in the frequency band $\mathrm{d}\omega$, associated with the wavenumber k of a plane wave of heading θ and propagating at a frequency ω, can be obtained by dividing the area $\mathrm{d}A$ by the area associated with each mode, that is

$$\mathrm{d}N(\omega,\theta) = \frac{S}{\pi^2}\,\mathrm{d}A = \frac{S}{\pi^2}\,k(\theta,\omega)\frac{\partial k(\theta,\omega)}{\partial\omega}\,\mathrm{d}\theta\,\mathrm{d}\omega. \tag{15}$$

Using Eq. (15), the average loss factor in Eq. (14) is thus given by

$$\overline{\eta}(\omega,T) = \frac{\int_0^{\pi} \eta(\omega,T,\theta)k(\omega,\theta)\dfrac{\partial k(\omega,\theta)}{\partial\omega}\,\mathrm{d}\theta}{\int_0^{\pi} k(\omega,\theta)\dfrac{\partial k(\omega,\theta)}{\partial\omega}\,\mathrm{d}\theta}. \tag{16}$$

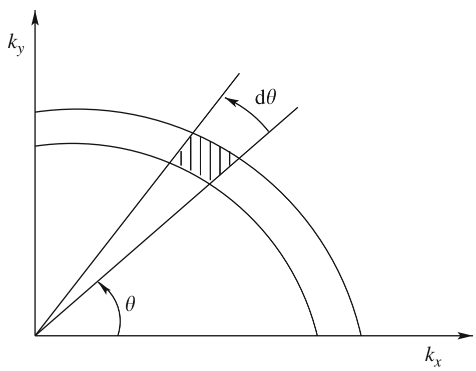

Fig. 3. Small segment of the wavenumber plane enclosed between the curves at ω and $\omega+\mathrm{d}\omega$ and the directions θ and $\theta+\mathrm{d}\theta$.

In Eq. (16), integration over the upper-half wavenumber plane should be considered for anisotropic systems, which have dispersion contour curves whose symmetry is of degree two. For isotropic components, integration can be reduced to the first quadrant of the wavenumber plane.

The methodology proposed in the present paper allows a straightforward numerical evaluation of the average loss factor according to Eq. (16). Once the cross-section characteristics and the material properties are known, this in fact merely involves solving a standard linear eigenvalue problem, i.e. Eq. (9), to evaluate the wavenumber $k(\omega,\theta)$ in polar coordinates, and using Eq. (10) or (12) to evaluate the global loss factor. The derivatives in Eq. (16) can be calculated numerically or using other techniques, e.g. [21].

3. Numerical examples

In this section various numerical examples are presented to illustrate the application of the WFE method for evaluating the global loss factor. The FE models are developed using the commercial FE software ANSYS, although any code could be used. Since flexural waves are generally the most important for response and noise radiation, only the loss factor related to the first flexural wave mode is discussed.

3.1. Aluminium plate with constrained layer damping treatment

As a first example, the WFE results for the flexural loss factor of an aluminium plate with an attached constrained layer damping treatment are presented. The results are compared with results obtained applying the theory in [1] and with results obtained by Ghinet and Atalla [10]. The WFE model is realised using 10 solid elements to discretise the bottom layer, two solid elements to discretise the viscoelastic core and one solid element to discretise the upper layer. The three cases studied correspond to cases X, Y and W in [1]. The results are shown in Fig. 4. It can be seen that the WFE results are in good agreement with the other results. At higher frequencies differences can be explained considering the a priori assumptions, which concern the variation of stress and strain through the thickness in [1,10]. Kerwin [1] estimated the loss factor by an analytical method under the assumption of the classical theory for a thin beam in bending vibration, while Ghinet and Atalla [10] used a discrete laminate method where the displacement field of any discrete layer was described by the Reissner–Mindlin theory.

3.2. Asymmetric angle-ply laminated sandwich plate

The WFE approach can be applied equally to laminates of arbitrary complexity, with an arbitrary number of layers. The example shown in this section concerns an asymmetric angle-ply laminated sandwich plate, whose skins comprise four sheets of 0.25 mm thick Graphite–Epoxy material. The stacking sequences of the bottom and the top skins are [45/−45/−45/45] and [−45/45/45/−45], respectively. The core is a 5 mm thick foam core. Material properties for the skins and the core are shown in Table 1. The FE model for the structure is realised using four solid elements for each skin and five solid elements to discretise the core, resulting in 13 elements. The reduced WFE model has therefore 42 DOFs. The length of the segment is L_x=L_y=L=0.001 m.

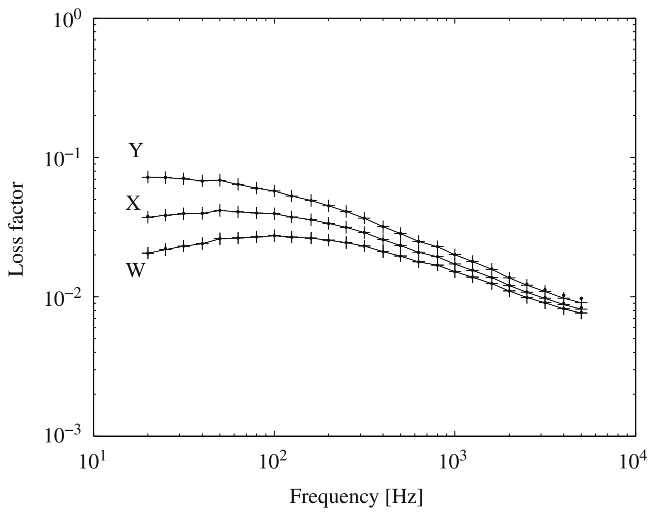

Fig. 4. Loss factor for the first flexural wave in an aluminium plate with CLD treatment: ··· WFE results; — theory of [10]; +++ theory of [1].

Table 1
Laminated sandwich plate: material properties.

Graphite-Epoxy			Foam core
E_x=119 GPa	E_y=8.67 GPa	E_z=8.67 GPa	E_x=0.18 GPa
G_{yz}=3.9 GPa	G_{xz}=5.18 GPa	G_{xy}=5.18 GPa	$\rho = 110\,\mathrm{kg/m^3}$
$v_{xy} = v_{xz} = 0.31$	$v_{yz} = 0.02$	$\rho = 1389\,\mathrm{kg/m^3}$	$v = 0.286$
$\eta_{xy} = 0.118\%$	$\eta_{xz} = 0.118\%$	$\eta_{yz} = 0.846\%$	$\eta = 10\%$
$\eta_x = 0.118\%$	$\eta_y = 0.620\%$	$\eta_z = 0.620\%$	

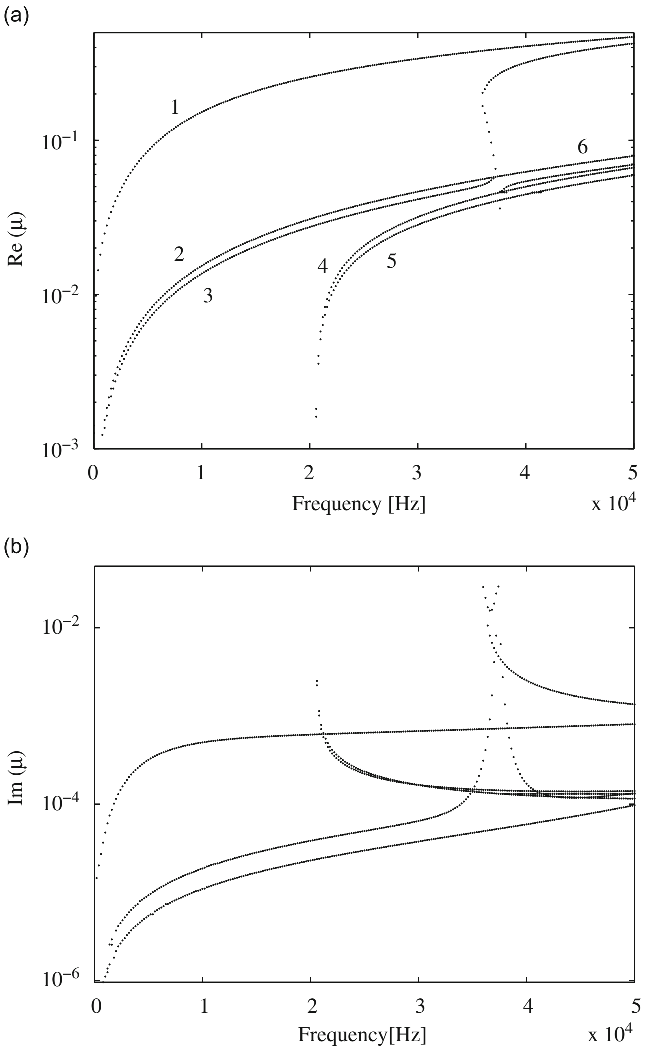

Fig. 5. Propagating waves in the laminated sandwich plate, $\theta = 0^\circ$; (a) real part of μ, (b) imaginary part of μ.

For harmonic propagating waves, material damping leads to a spatial decay of waves propagating in the structure: the frequency is real and positive while the wavenumber is complex – the imaginary part of the wavenumber describing the decay. Fig. 5 shows the real and imaginary parts of the propagation constant, $\mu = kL$, versus frequency for waves propagating in the $\theta = 0^\circ$ direction. Up to 3.6 kHz, the first three branches in Fig. 5(a) represent, respectively, the first quasi-flexural, quasi-shear and quasi-extensional waves propagating in the laminated sandwich. At higher frequencies, the dispersion characteristics become very complicated, involving coupling between the various wave modes, veering and so on. Fig. 5(a) shows that further propagating branches cut-off as the frequency increases, branches 4, 5, and 6 in Fig. 5(a), which involve higher order modes across the thickness of the plate. Fig. 5(b) shows the attenuation of propagating waves in terms of the imaginary part of the non-dimensional wavenumber $\mu = kL$.

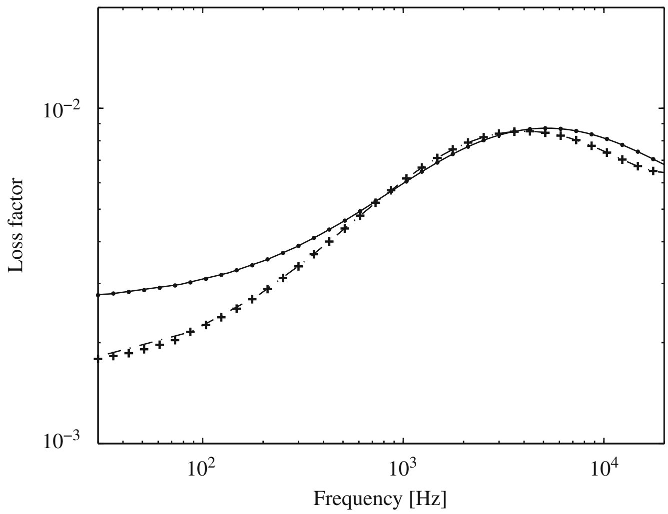

Fig. 6. Loss factor for the first flexural wave in the laminated sandwich plate as a function of frequency. $\theta = 0^{\circ}$: ··· obtained from Eq. (10) and — obtained from Eq. (12); $\theta = 45^{\circ}$: +++ obtained from Eq. (10) and - · - · - obtained from Eq. (12).

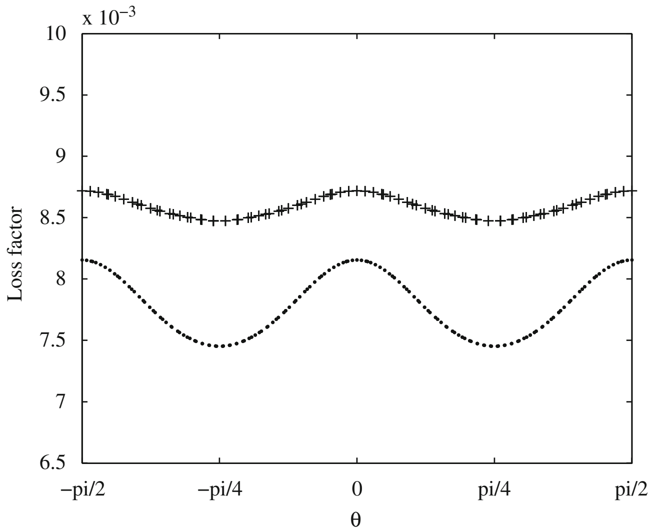

Fig. 7. Loss factor for the first flexural wave in the laminated sandwich plate as a function of the propagation direction: +++ 5 kHz and ··· 10 kHz.

The global loss factor for the first quasi-flexural wave, i.e. the first branch in Fig. 5(a), is shown in Fig. 6. As an example, two propagation directions are considered: $\theta = 0^{\circ}$ and 45°. The figure shows the results obtained by solving Eqs. (10) and (12). There are very small differences which can be attributed to very small differences in the assumed response, i.e. harmonic in time or harmonic in space. It can be noticed that the global loss factor of the sandwich panel increases with frequency up to about 4–5 kHz, and then slightly decreases. Similar results for sandwich configurations were obtained, for example, by Taylor and Nayfeh in [22], and by Ghinet and Atalla in [10]. Fig. 7 shows the loss factor as a function of the propagation direction θ. The two branches in Fig. 7 are obtained by solving Eq. (12) for the first flexural wave propagating at 5 and 10 kHz. Although the ability of the Epoxy matrix to dissipate energy is slight, it can be observed that the damping is affected by the direction of propagation.

3.3. *Laminate plates with viscoelastic layer*

This section considers three different cases in which the viscoelastic layer is sandwiched between two laminate face plates. Table 2 shows the stacking sequences and thicknesses of the three plates. The density and the Poisson ratio of the damping material are $\rho = 110\,\text{kg/m}^3$ and $\nu = 0.48$, while the frequency dependent shear modulus and loss factor are

Table 2
Stacking sequence and thickness.

	Bottom face plate	Viscoelastic layer (core)	Upper face plate
Case 1	$[45/-45]_S$ 2 mm	[0] 0.2 mm	[45] 0.25 mm
Case 2	$[45/-45]_S$ 2 mm	[0] 0.2 mm	[−45] 0.25 mm
Case 3	[45/−45] 1 mm	[0] 0.2 mm	[−45/45] 1 mm

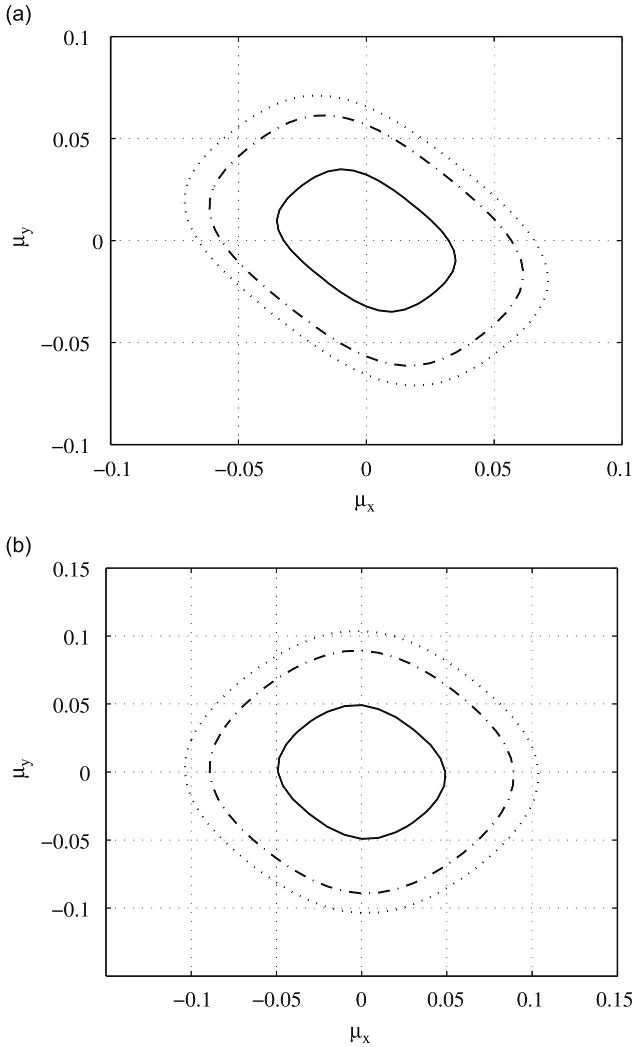

Fig. 8. Dispersion contour curves for the first quasi-flexural wave: — 500 Hz; - · - · - 1.5 kHz; · · · 2 kHz; (a) case 1, (b) case 3.

extracted from [1], and approximated by cubic functions of frequency. The contribution to the energy dissipation of the fibre reinforced materials is also taken into account according to the material properties given in Table 1.

Dispersion contour curves provide information of various forms. For example they allow determination of the direction of propagation and evaluation of the modal density. In particular, the latter information is here used to obtain the average loss factor as described in Section 2.3. Fig. 8 shows, as an example, the dispersion contour curves in the plane (μ_x,μ_y), $\mu_x = k_x L_x$ and $\mu_y = k_y L_y$, for the first quasi-flexural wave at 500 Hz, 1.5 and 2 kHz for the laminate configurations corresponding to cases 1 and 3. Dispersion contours for case 2 are not given since they are almost identical to the one obtained for case 1. Figs. 8(b) and (a) show, respectively that, at the frequency considered, the flexural behaviour of laminate 3 is quasi-orthotropic (the dispersion contours have approximately two axes of symmetry), while that of laminate 1 (and laminate 2) is anisotropic.

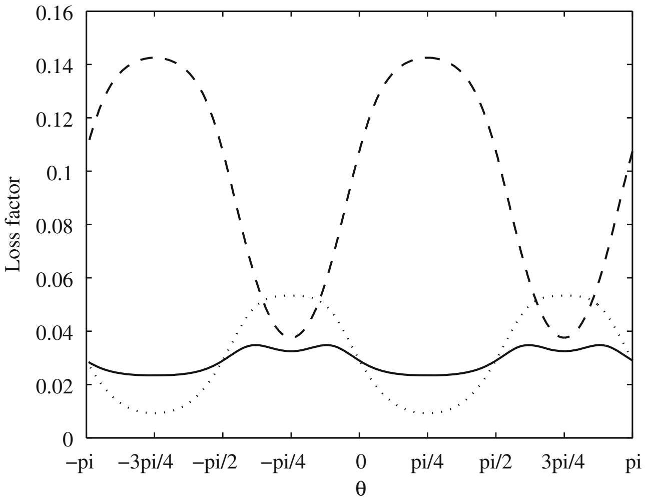

Fig. 9. Global loss factor for the first flexural wave at 1.5 kHz as a function of the heading direction θ: — case 1; ⋯ case 2; ---- case 3.

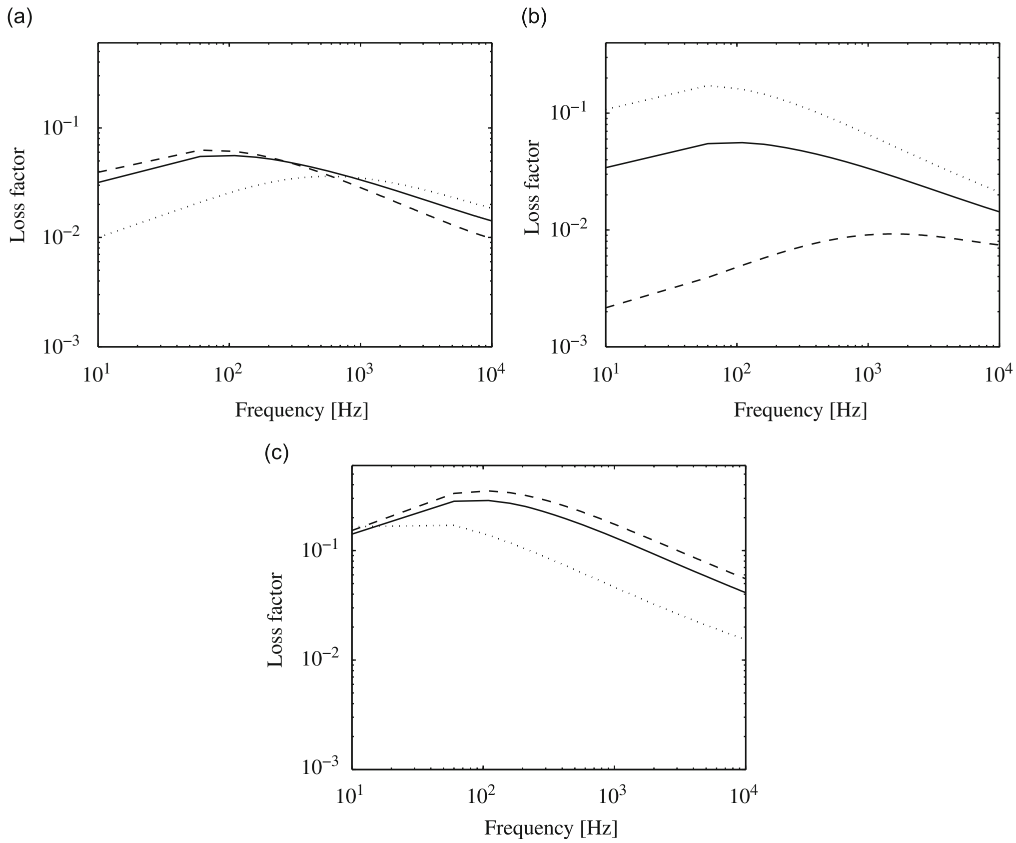

Fig. 10. Global loss factor for the first flexural wave as a function of frequency: — $\theta=0$; ---- $\theta=\pi/4$; ⋯ $\theta=3\pi/4$; (a) case 1, (b) case 2, (c) case 3.

The global loss factors of the laminated panels are shown in Figs. 9 and 10. Fig. 9 shows the global loss factor for the first flexural wave at 1.5 kHz as a function of the propagation direction, while Fig. 10 shows the global loss as a function of the frequency for $\theta=0,\pi/4,3\pi/4$. It can be noticed that the loss factor has a maximum around 100 Hz. It can be also observed that, due to laminate characteristics and the spatially non-uniform distribution of shearing in the viscoelastic layer, the damping is clearly affected by the direction of propagation.

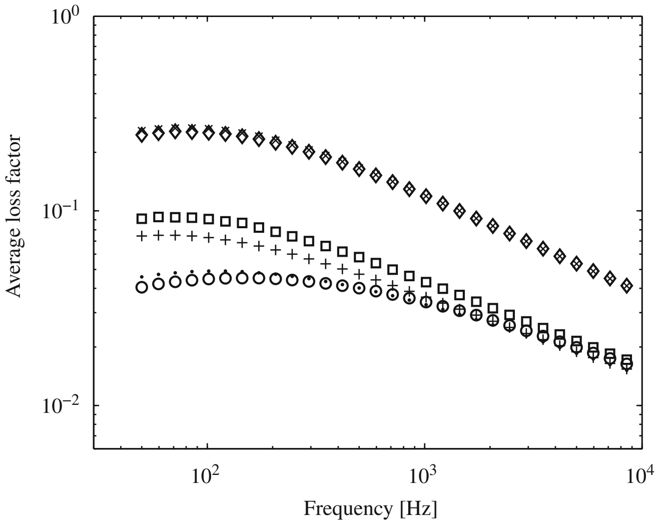

Fig. 11. Average loss factor: ○ case 1, □ case 2, ◇ case 3 obtained from Eq. (16); ··· case 1, +++ case 2, × × × case 3 obtained from Eq. (13).

The average loss factors for the three panels are given in Fig. 11. These results are obtained by evaluating the derivatives and the integrals in Eq. (16) numerically. Comparison with the results obtained evaluating the average as in Eq. (13) is also shown. Fig. 11 shows that the configuration in which the damping layer is sandwiched between two face-plates having the same thickness, i.e. case 3, is the most effective in dissipating flexural energy. The symmetric configuration in fact maximises interlaminar shear in the damping layer and hence yields higher loss factor.

4. Conclusion

Dispersion and damping characteristics of viscoelastic laminate plates were investigated using a wave finite element method for two-dimensional structures. The damping performances of viscoelastic structures were evaluated using the global loss factor in a wave context approach, when the viscoelasticity is modelled by considering a complex stiffness matrix formulation. The global loss factor was predicted considering both a wave formulation of the modal strain energy method and a formulation obtained assuming no spatial decay but a temporal decay. A formulation for evaluating the average loss factor in the general case of an anisotropic component was also proposed. Finally various examples were presented. They included damped laminated panels of different constructions for which analytical approaches are extremely complicated. The method provides a fast, efficient and automated technique for predicting dispersion, attenuation and damping in composite structures for which inherent material damping is not negligible.

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